Hypothetical/Conditional Propositions

Conditional Propositions are propositions that express conditional relations between two states of affairs. Examples are “I won’t come over if there’s a football game on,” “whenever I eat seafood, I throw up,” “I’ll come over unless my bookie breaks my kneecaps,” “I’ll only win the race if I enter it,” etc., where the indicator words for conditional propositions are in bold.

There are two different types of conditions, necessary and sufficient. “Something (let’s call it A) is a necessary condition for something else (B)” means that you have to have A to have B, or, without A, you can’t have B. Other ways to say this are:
1. B only if A (e.g.: I’ll pass the test only if I study)
2. If not A then not B (e.g.: If my battery is not charged then my car won’t start)
3. If B then A (e.g.: If I’m running a marathon then I’m alive)

“P is a sufficient condition for Q” means that P is enough to get you Q, or, having P means that you have Q. Other ways to say this are:
4. Q if P (You’ll die if you jump out of this ‘plane)
5. If P then Q (If you jump out of this ‘plane then you’ll die)
6. Q unless not P (I’ll go to the movies unless I’m not well)

NOTICE: If A is necessary for B, then B is sufficient for A (and vice versa)

Any conditional proposition can be converted into standard form for hypothetical propositions. Standard form is:

IF (antecedent) THEN (consequent)

Of 1-6 above, 2, 3 and 5 are already in standard form. The others can be re-written as follows:
1. If B then A (If I pass the test then I studied)
4. If P then Q (If you jump out of this ‘plane then you’ll die - same as 5)
6. If P then Q (If I’m well then I’ll go to the movies)

From the above we can see that the antecedent in any hypothetical proposition in standard form is sufficient for the consequent, and the consequent is necessary for the antecedent. This means that:

IF A THEN B says exactly the same as IF NOT B THEN NOT A

(for example, “If I shoot you, then you’re dead” means that shooting is sufficient for death [and thus death is necessary for shooting]. “If you’re not dead then I didn’t shoot you” means that death is necessary for shooting [and thus shooting is sufficient for death])

There are four classic two-premise arguments that have hypothetical propositions as the first premise (i.e., mixed hypothetical syllogisms), two of which are valid, and two of which are not. The names of the arguments describe what the non-hypothetical premises do to the hypothetical premises. So, for example:

If my battery is dead then my car won’t start  \( B \rightarrow \neg S \)
My battery IS dead \( B \)
My car won’t start \( \neg S \)

This is affirming the antecedent (also called modus ponens), because the second premise states that the antecedent of the hypothetical premise is true. The other three types are affirming the consequent, denying the antecedent and denying the consequent.

Which of these are valid, and which are not? The easiest one to tell is affirming the antecedent, like the example given above, which is clearly valid. The reason it is valid, we can now see, is because the hypothetical premise says that the antecedent is sufficient for the consequent, and the other premise says that the antecedent is true, so this means (from the definition of sufficient condition) that the consequent must follow. This gives us a clue what the other valid syllogism will be: remember that if something (having a charged battery) is necessary for something else (your car starting) then you know that without the first thing you can’t have the second. Remember also that the consequent in a hypothetical proposition is necessary for the antecedent. Put these two facts together and you know that if the consequent is not true then the antecedent can’t be true. So the other valid hypothetical syllogism is denying the consequent (also called modus tollens):

If my car starts then my battery is NOT dead  \( S \rightarrow \neg B \)
My battery IS dead \( B \)
My car won’t start \( \neg S \)