1

Functions and Graphs

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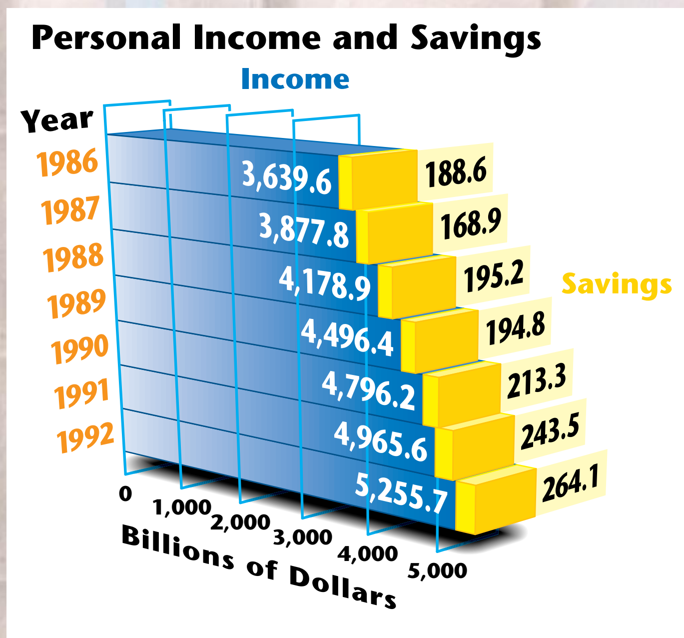
Chapter 1 Group Activity: Introduction to Regression Analysis

Chapter 1 Review

Application

The data shown in the chart represent the yearly totals for personal income and savings in the United States. Use linear regression on a graphing utility to predict personal income when personal savings grow to $300 billion.



The function concept is one of the most important ideas in mathematics. The study of either the theory or the applications of mathematics beyond the most elementary level requires a firm understanding of functions and their graphs. The first two sections of this chapter are concerned with basic graphing techniques, including point-by-point plotting, graphing circles, and using an electronic graphing device such as a graphing calculator or a computer. In the remaining sections, we introduce the important concept of a function, discuss basic properties of functions and their graphs, and examine specific types of functions. Much of the remainder of this book is concerned with applying the ideas introduced in this chapter to a variety of different types of functions, as is evidenced by the chapter titles following this chapter (check the table of contents). Efforts made to understand and use the function concept correctly from the beginning will be rewarded many times in this course and in most future courses that involve mathematics.

Preparing for This Chapter

Before getting started on this chapter, review the following concepts:

Set Notation (Appendix A, Section A-1)

Polynomials (Appendix A, Sections A-2 and A-3)

Rational Expressions (Appendix A, Section A-4)

Square Root Radicals (Appendix A, Section A-7)

Interval Notation (Appendix A, Section A-8)

Pythagorean Theorem (Appendix D)

Section 1-1 | Cartesian Coordinate System

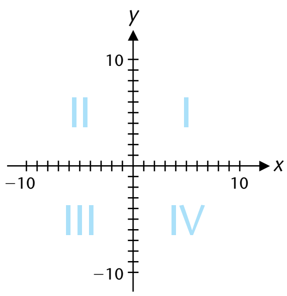
- Cartesian Coordinate System
- Graphing: Point by Point
- Distance between Two Points
- Circles

Analytic geometry is concerned with the relationship between geometric forms, such as circles and lines, and algebraic forms, such as equations and inequalities. The key to this relationship is the Cartesian coordinate system, named after the French mathematician and philosopher René Descartes (1596–1650) who was the first to combine the study of algebra and geometry into a single discipline. In this section, we develop some of the basic tools used in analytic geometry and apply these tools to the graphing of equations and to the derivation of the equation of a circle.

Cartesian Coordinate System

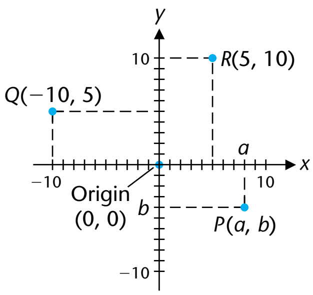
Just as a real number line establishes a one-to-one correspondence between the points on a line and the elements in the set of real numbers, we can form a **real plane** by establishing a one-to-one correspondence between the points in a plane and elements in the set of all ordered pairs of real numbers. This can be done by means of a Cartesian coordinate system.

FIGURE 1
Cartesian coordinate system.



Recall that to form a **Cartesian** or **rectangular coordinate system,** we select two real number lines, one horizontal and one vertical, and let them cross through their origins as indicated in Figure 1. Up and to the right are the usual choices for the positive directions. These two number lines are called the **horizontal axis** and the **vertical axis,** or together, the **coordinate axes.** The horizontal axis is usually referred to as the ***x* axis** and the vertical axis as the ***y* axis,** and each is labeled accordingly. Other labels may be used in certain situations. The coordinate axes divide the plane into four parts called **quadrants,** which are numbered counterclockwise from I to IV (see Fig. 1).

FIGURE 2
Coordinates in a plane.



Now we want to assign *coordinates* to each point in the plane. Given an arbitrary point P in the plane, pass horizontal and vertical lines through the point (Fig. 2). The vertical line will intersect the horizontal axis at a point with coordinate a, and the horizontal line will intersect the vertical axis at a point with coordinate b. These two numbers written as the ordered pair (a, b) form the **coordinates** of the point P. The first coordinate a is called the **abscissa** of P; the second coordinate b is called the **ordinate** of P. The abscissa of Q in Figure 2 is -10, and the ordinate of Q is 5. The coordinates of a point can also be referenced in terms of the axis labels. The ***x* coordinate** of R in Figure 2 is 5, and the ***y* coordinate** of R is 10. The point with coordinates $(0, 0)$ is called the **origin.**

The procedure we have just described assigns to each point P in the plane a unique pair of real numbers (a, b). Conversely, if we are given an ordered pair of real numbers (a, b), then, reversing this procedure, we can determine a unique point P in the plane. Thus:

There is a one-to-one correspondence between the points in a plane and the elements in the set of all ordered pairs of real numbers.

This is often referred to as the **fundamental theorem of analytic geometry.** Given any set of ordered pairs S, the **graph** of S is the set of points in the plane corresponding to the ordered pairs in S.

Graphing: Point by Point

The fundamental theorem of analytic geometry enables us to look at algebraic forms geometrically and to look at geometric forms algebraically. We begin by considering an algebraic form, an equation in two variables:

$$y = x^2 - 4 \tag{1}$$

A **solution** to equation (1) is an ordered pair of real numbers (a, b) such that

$$b = a^2 - 4$$

The **solution set** of equation (1) is the set of all these ordered pairs. More formally,

$$\text{Solution set of equation (1): } \{(x, y) \mid y = x^2 - 4\} \tag{2}$$

To find a solution for equation (1) we simply replace x with a number and calculate the value of y. For example, if $x = 2$, then $y = 2^2 - 4 = 0$, and the ordered pair (2, 0) is a solution. Similarly, if $x = -3$, then $y = (-3)^2 - 4 = 5$, and the ordered pair $(-3, 5)$ is also a solution of equation (1). In fact, any real number substituted for x in equation (1) will produce a solution to the equation. Thus, the solution set shown in (2) must have an infinite number of elements. We now use a rectangular coordinate system to provide a geometric representation of this set.

The **graph of an equation** is the graph of its solution set. To *sketch the graph of an equation,* we plot enough points from its solution set so that the total graph is apparent and then connect these points with a smooth curve, proceeding from left to right. This process is called **point-by-point plotting.**

EXAMPLE 1

Graphing an Equation Using Point-by-Point Plotting

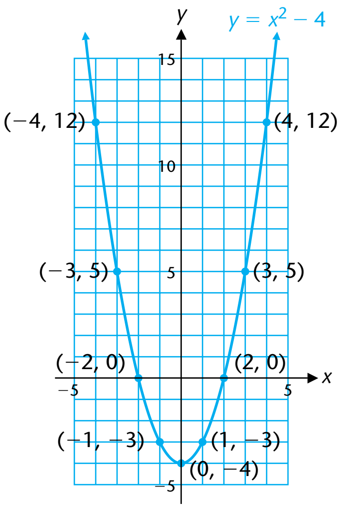
Sketch a graph of $y = x^2 - 4$.

Solution

We make up a table of solutions—ordered pairs of real numbers that satisfy the given equation.

x	-4	-3	-2	-1	0	1	2	3	4
y	12	5	0	-3	-4	-3	0	5	12

FIGURE 3



After plotting these solutions, if there are any portions of the graph that are unclear, we plot additional points until the shape of the graph is apparent. Then we join all these plotted points with a smooth curve, as shown in Figure 3. Arrowheads are used to indicate that the graph continues beyond the portion shown here with no significant changes in shape.

The resulting figure is called a *parabola.* Notice that if we fold the paper along the y axis, the right side will match the left side. We say that the graph is *symmetric with respect to the y axis* and call the y axis the *axis of the parabola.* More will be said about parabolas later in the text.

MATCHED PROBLEM 1*

Sketch a graph of $y = 8 - x^2$ using point-by-point plotting.

EXAMPLE 2

Graphing an Equation Using Point-by-Point Plotting

Sketch a graph of $y = \sqrt{x}$.

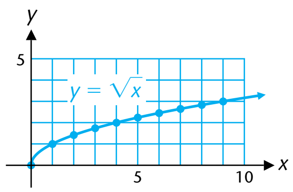
Solution

Proceeding as before, we make up a table of solutions:

x	0	1	2	3	4	5	6	7	8	9
y	0	1	$\sqrt{2} \approx 1.4$	$\sqrt{3} \approx 1.7$	2	$\sqrt{5} \approx 2.2$	$\sqrt{6} \approx 2.4$	$\sqrt{7} \approx 2.6$	$\sqrt{8} \approx 2.8$	3

*Answers to matched problems in a given section are found near the end of the section, before the exercise set.

FIGURE 4



For graphing purposes, the irrational numbers in the table were evaluated on a calculator and rounded to one decimal place. Plotting these points and connecting them with a smooth curve produces the graph in Figure 4.

Notice that we did not include any negative values of x in the table. If x is a negative real number, then $\sqrt{x}$ is not a real number. Since the coordinates of a point in a rectangular coordinate system must be real numbers, *when graphing an equation, we consider only those values of the variables that produce real solutions to the equation.* We will have more to say about numbers of the form $\sqrt{x}$, where x is negative, later in this book.

MATCHED PROBLEM 2

Sketch a graph of $y = 4 - \sqrt{x}$.

Explore/Discuss 1

To graph the equation $y = -x^3 + 2x$, we use point-by-point plotting to obtain the graph in Figure 5.

x	y
-1	-1
0	0
1	1

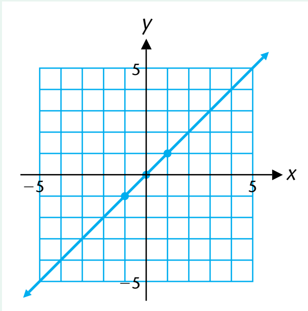


FIGURE 5

(A) Do you think this is the correct graph of the equation? If so, why? If not, why?

(B) Add points on the graph for $x = -2, -0.5, 0.5$, and 2.

(C) Now, what do you think the graph looks like? Sketch your version of the graph, adding more points as necessary.

(D) Write a short statement explaining any conclusions you might draw from parts (A), (B), and (C).

As Explore/Discuss 1 illustrates, sometimes it can be difficult to determine the apparent shape of a graph by simply plotting a few points. One of the major objectives of this book is to develop mathematical tools that will help us analyze graphs.

The use of graphs to illustrate relationships between quantities is commonplace. Estimating the coordinates of points on a graph provides specific examples of this relationship, even if no equation for the graph is available. The next example illustrates this process.

EXAMPLE 3

Ozone Levels

The ozone level is measured in parts per billion (ppb). The ozone level during a 12-hour period in a suburb of Milwaukee, Wisconsin, on a particular

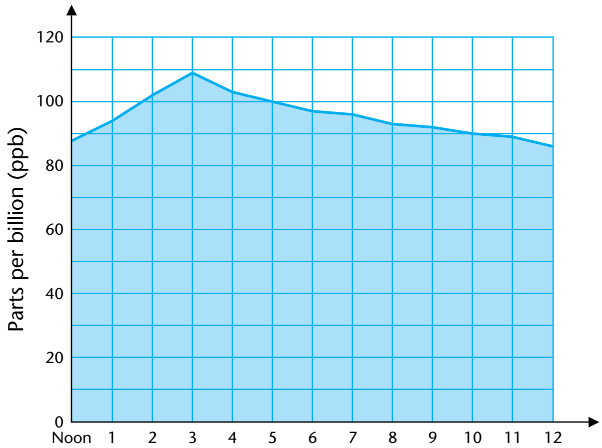
summer day is given in Figure 6. Use this graph to estimate the following ozone levels to the nearest integer and times to the nearest quarter hour:

(A) The ozone level at 6 P.M.

(B) The highest ozone level and the time when it occurs.

(C) The time(s) when the ozone level is 90 ppb.

FIGURE 6



Source: Wisconsin Department of Natural Resources.

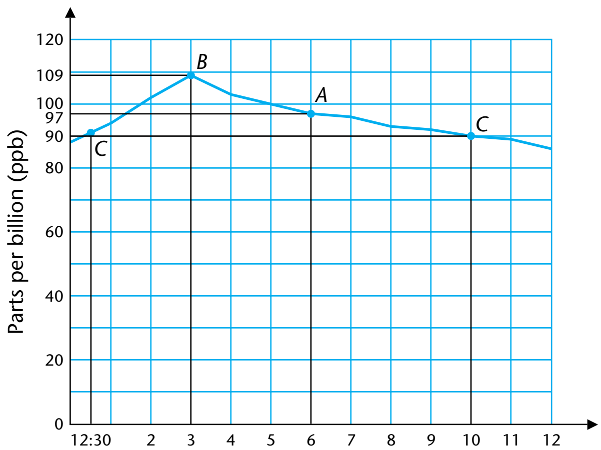
Solutions

(A) The ordinate of the point on the graph with abscissa 6 is approximately 97 ppb (see Fig. 7).

(B) The highest ozone level is approximately 109 ppb at 3 P.M.

(C) The ozone level is 90 ppb at about 12:30 P.M. and again at 10 P.M.

FIGURE 7



MATCHED PROBLEM 3

Use the graph in Figure 6 to estimate the following ozone level to the nearest integer and times to the nearest quarter hour.

(A) The ozone level at 7 P.M.

(B) The time(s) when the ozone level is 100 ppb.

Distance between Two Points

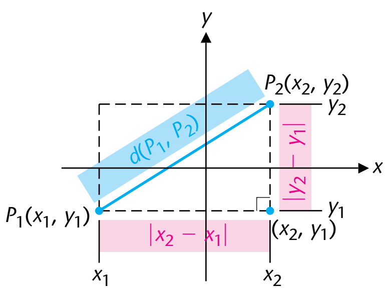
Analytic geometry is concerned with two basic problems:

1. Given an equation, find its graph.
2. Given a figure (line, circle, parabola, ellipse, etc.) in a coordinate system, find its equation.

So far we have concentrated on the first problem. We now introduce a basic tool that is used extensively in solving the second problem. This basic tool is the *distance-between-two-points formula*, which is easily derived using the Pythagorean theorem (see Appendix D). Let $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ be two points in a rectangular coordinate system and let $d(P_1, P_2)$ represent the distance between these two points. Then referring to Figure 8, we see that

$$\begin{aligned}[d(P_1, P_2)]^2 &= |x_2 - x_1|^2 + |y_2 - y_1|^2 \\ &= (x_2 - x_1)^2 + (y_2 - y_1)^2 \qquad \text{Since } |N|^2 = N^2.\end{aligned}$$

FIGURE 8
Distance between two points.



Thus:

THEOREM 1

DISTANCE BETWEEN $P_1(x_1, y_1)$ AND $P_2(x_2, y_2)$

$$d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

EXAMPLE 4

Using the Distance-between-Two-Points Formula

Find the distance between the points $(-3, 5)$ and $(-2, -8)$.*

Solution

It doesn't matter which point we designate as P_1 or P_2 because of the squaring in the formula. Let $(x_1, y_1) = (-3, 5)$ and $(x_2, y_2) = (-2, -8)$. Then,

$$\begin{aligned}d &= \sqrt{[(-2) - (-3)]^2 + [(-8) - 5]^2} \\ &= \sqrt{(-2 + 3)^2 + (-8 - 5)^2} = \sqrt{1^2 + (-13)^2} = \sqrt{1 + 169} = \sqrt{170}\end{aligned}$$

*We often speak of the point (a, b) when we are referring to the point with coordinates (a, b). This shorthand, though not accurate, causes little trouble, and we will continue the practice.

Notice that if we choose $(x_1, y_1) = (-2, -8)$ and $(x_2, y_2) = (-3, 5)$, then

$$d = \sqrt{[(-3) - (-2)]^2 + [5 - (-8)^2]} = \sqrt{1 + 169} = \sqrt{170}$$

so it doesn't matter which point we designate as P_1 or P_2.

MATCHED PROBLEM 4

Find the distance between the points $(6, -3)$ and $(-7, -5)$.

Circles

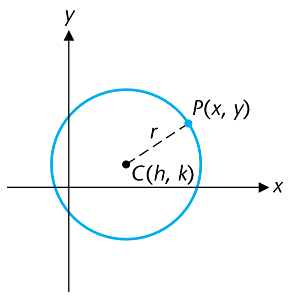
The distance-between-two-points formula would still be helpful if its only use were to find actual distances between points, such as in Example 4. However, its more important use is in finding equations of figures in a rectangular coordinate system. We will use it to derive the standard equation of a circle. We start with a coordinate-free definition of a circle.

DEFINITION 1

CIRCLE

A **circle** is the set of all points in a plane equidistant from a fixed point. The fixed distance is called the **radius,** and the fixed point is called the **center.**

FIGURE 9
Circle.



Let's find the equation of a circle with radius r ($r > 0$) and center C at (h, k) in a rectangular coordinate system (Fig. 9). The circle consists of all points $P(x, y)$ satisfying $d(P, C) = r$; that is, all points satisfying

$$\sqrt{(x - h)^2 + (y - k)^2} = r \quad r > 0$$

or, equivalently,

$$(x - h)^2 + (y - k)^2 = r^2 \quad r > 0$$

THEOREM 2

STANDARD EQUATION OF A CIRCLE

Circle with radius r and center at (h, k):

$$(x - h)^2 + (y - k)^2 = r^2 \quad r > 0$$

EXAMPLE 5

Equations and Graphs of Circles

Find the equation of a circle with radius 4 and center at:

(A) $(0, 0)$ (B) $(-3, 6)$

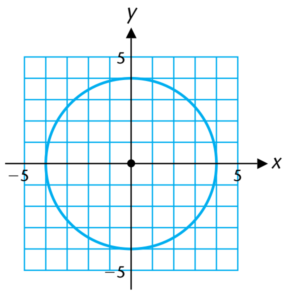
Graph each equation.

Solutions

(A) $(h, k) = (0, 0)$ and $r = 4$:

$$x^2 + y^2 = r^2$$
$$x^2 + y^2 = 4^2$$
$$x^2 + y^2 = 16$$

To graph the equation, locate the center at the origin and draw a circle of radius 4 (Fig. 10).



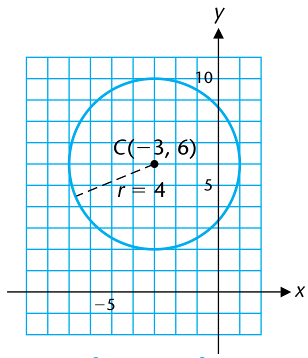
$x^2 + y^2 = 16$

FIGURE 10

(B) $(h, k) = (-3, 6)$ and $r = 4$:

$$(x - h)^2 + (y - k)^2 = r^2$$
$$[x - (-3)]^2 + (y - 6)^2 = 4^2$$
$$(x + 3)^2 + (y - 6)^2 = 16$$

To graph the equation, locate the center $C(-3, 6)$ and draw a circle of radius 4 (Fig. 11).



$(x + 3)^2 + (y - 6)^2 = 16$

FIGURE 11

MATCHED PROBLEM 5

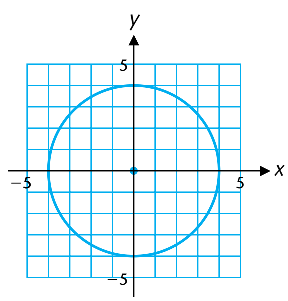
Find the equation of a circle with radius 3 and center at:

(A) $(0, 0)$ (B) $(3, -2)$

Graph each equation.

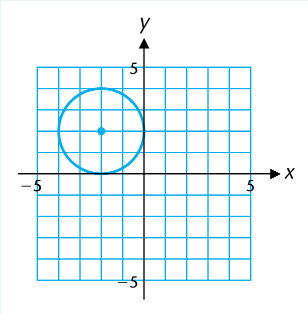
Explore/Discuss 2

Each of the following statements is false. Indicate why, then modify each equation to make the statement true.

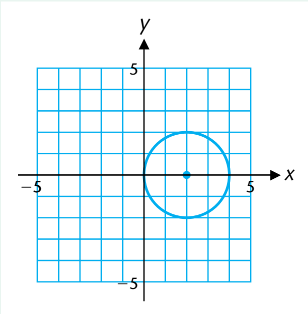


is the graph of $x^2 + y^2 = 4$.

Explore/Discuss 2 continued



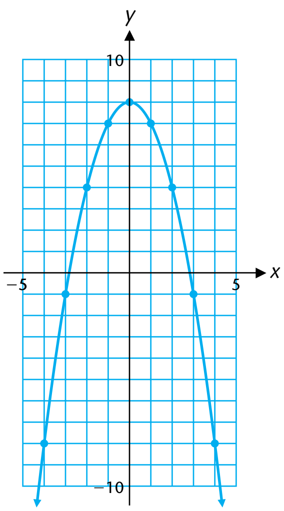
is the graph of $(x-2)^2 + (y+2)^2 = 4$.



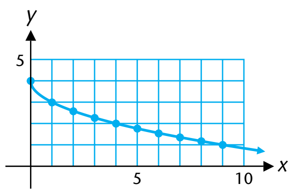
is the graph of $x^2 + (y-2)^2 = 4$.

Answers to Matched Problems

1.



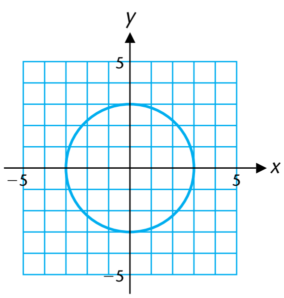
2.



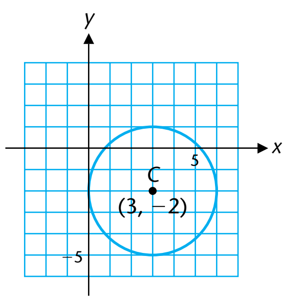
3. (A) 96 ppb (B) 1:45 P.M. and 5 P.M.

4. $d = \sqrt{173}$

5. (A) $x^2 + y^2 = 9$



(B) $(x-3)^2 + (y+2)^2 = 9$



EXERCISE 1-1

A

In Problems 1–4, plot the given points in a rectangular coordinate system.

1. $(5, 0), (3, -2), (-4, 2), (4, 4)$

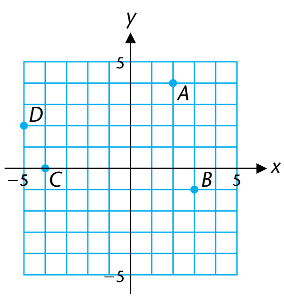
2. $(0, 4), (-3, 2), (5, -1), (-2, -4)$

3. $(0, -2), (-1, -3), (4, -5), (-2, 1)$

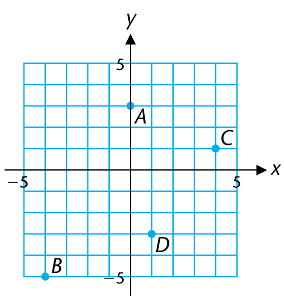
4. $(-2, 0), (3, 2), (1, -4), (-3, 5)$

In Problems 5–8, find the coordinates of points A, B, C, and D.

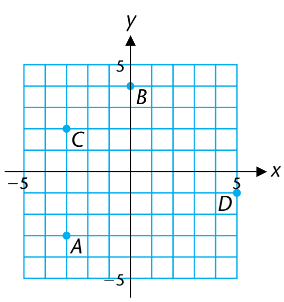
5.



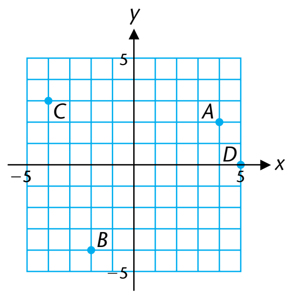
6.



7.



8.



Find the distance between the indicated points in Problems 9–12. Leave the answer in radical form.

9. $(-6, -4), (3, 4)$

10. $(-5, 4), (6, -1)$

11. $(6, 6), (4, -2)$

12. $(5, -3), (-1, 4)$

In Problems 13–20, write the equation of a circle with the indicated center and radius.

13. $C(0, 0), r = 7$

14. $C(0, 0), r = 5$

15. $C(2, 3), r = 6$

16. $C(5, 6), r = 2$

17. $C(-4, 1), r = \sqrt{7}$

18. $C(-5, 6), r = \sqrt{11}$

19. $C(-3, -4), r = \sqrt{2}$

20. $C(4, -1), r = \sqrt{5}$

B

For each equation in Problems 21–26, make up a table of solutions using $x = -3, -2, -1, 0, 1, 2$, and 3. Plot these solutions and graph the equation.

21. $y = x + 1$

22. $y = 2 - x$

23. $y = x^2 - 5$

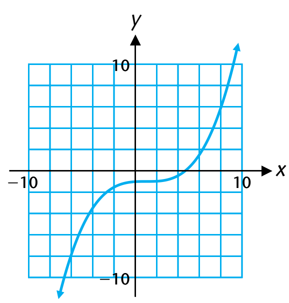
24. $y = 4 - x^2$

25. $y = 3 + x - 0.5x^2$

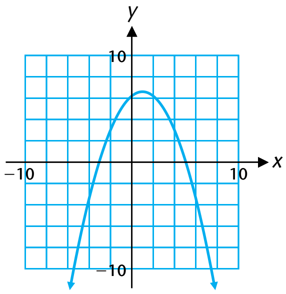
26. $y = 4 - x - 0.5x^2$

In Problems 27–30, use the graph to estimate to the nearest integer the missing coordinates of the indicated points. (Be sure you find all possible answers.)

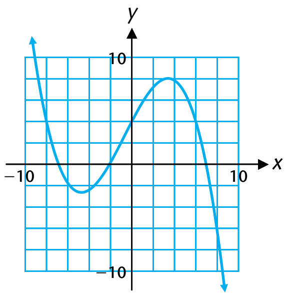
27. (A) $(8, ?)$ (B) $(-5, ?)$ (C) $(0, ?)$
(D) $(?, 6)$ (E) $(?, -5)$ (F) $(?, 0)$



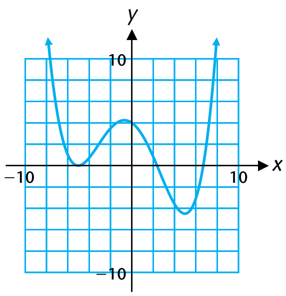
28. (A) (3, ?) (B) $(-5, ?)$ (C) (0, ?)
(D) (?, 3) (E) $(?, -4)$ (F) (?, 0)



29. (A) (1, ?) (B) $(-8, ?)$ (C) (0, ?)
(D) $(?, -6)$ (E) (?, 4) (F) (?, 0)



30. (A) (6, ?) (B) $(-6, ?)$ (C) (0, ?)
(D) $(?, -2)$ (E) (?, 1) (F) (?, 0)



In Problems 31 and 32, determine whether the given points are vertices of a right triangle. (Recall, a triangle is a right triangle if and only if the square of the longest side is equal to the sum of the squares of the shorter sides.)

31. $(-3, 2), (1, -2), (8, 5)$ **32.** $(-4, -1), (0, 7), (6, -6)$

Find the perimeter (to two decimal places) of the triangle with the vertices indicated in Problems 33 and 34.

33. $(-3, 1), (1, -2), (4, 3)$ **34.** $(-2, 4), (3, 1), (-3, -2)$

In Problems 35–42, graph each equation using point-by-point plotting.

35. $y = x^{1/3}$ **36.** $y = x^{2/3}$

37. $y = x^3$ **38.** $y = x^4$

39. $y = \sqrt{x - 1}$ **40.** $y = \sqrt{5 - x}$

41. $y = \sqrt{1 + x^2}$ **42.** $y = x\sqrt{1 + x^2}$

43. (A) Graph the triangle with vertices $A(1, 1)$, $B(7, 2)$, and $C(4, 6)$.

(B) Now graph the triangle with vertices $A'(1, -1)$, $B'(7, -2)$, and $C'(4, -6)$ in the same coordinate system.

(C) How are these two triangles related? How would you describe the effect of changing the sign of the y coordinate of all the points on a graph?

44. (A) Graph the triangle with vertices $A(1, 1)$, $B(7, 2)$, and $C(4, 6)$.

(B) Now graph the triangle with vertices $A'(-1, 1)$, $B'(-7, 2)$, and $C'(-4, 6)$ in the same coordinate system.

(C) How are these two triangles related? How would you describe the effect of changing the sign of the x coordinate of all the points on a graph?

45. (A) Graph the triangle with vertices $A(1, 1)$, $B(7, 2)$, and $C(4, 6)$.

(B) Now graph the triangle with vertices $A'(-1, -1)$, $B'(-7, -2)$, and $C'(-4, -6)$ in the same coordinate system.

(C) How are these two triangles related? How would you describe the effect of changing the signs of the x and y coordinates of all the points on a graph?

46. (A) Graph the triangle with vertices $A(1, 2)$, $B(1, 4)$, and $C(3, 4)$.

(B) Now graph $y = x$ and the triangle obtained by reversing the coordinates for each vertex of the original triangle: $A'(2, 1)$, $B'(4, 1)$, $C'(4, 3)$.

(C) How are these two triangles related? How would you describe the effect of reversing the coordinates of each point on a graph?

C

47. Use the distance-between-two-points formula to show that the point

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

is the **midpoint** of the line segment joining (x_1, y_1) and (x_2, y_2).

48. Use the midpoint formula from Problem 47 to find the midpoint of the line segment joining $(-3, 2)$ and $(5, -2)$.

Find the equation of a circle that has a diameter with the end points given in Problems 49 and 50. [Hint: See Problem 47.]

49. $(7, -3), (1, 7)$

50. $(-3, 2), (7, -4)$

51. Find the equation of a circle with center $(2, 2)$ whose graph passes through the point $(3, -5)$.

52. Find the equation of a circle with center $(-5, 4)$ whose graph passes through the point $(2, -3)$.

APPLICATIONS

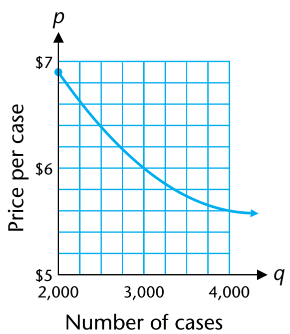
53. Price and Demand. The quantity of a product that consumers are willing to buy during some period of time depends on its price. The price p and corresponding weekly demand q for a particular brand of diet soda in a city are shown in the figure. Use this graph to estimate the following demands to the nearest 100 cases.

(A) What is the demand when the price is \$6.00 per case?

(B) Does the demand increase or decrease if the price is increased to \$6.30 per case? By how much?

(C) Does the demand increase or decrease if the price is decreased to \$5.70? By how much?

(D) Write a brief description of the relationship between price and demand illustrated by this graph.



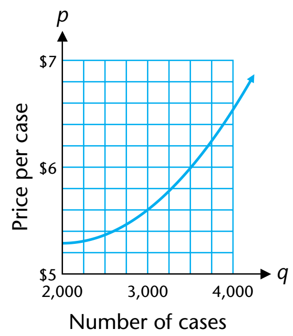
54. Price and Supply. The quantity of a product that suppliers are willing to sell during some period of time depends on its price. The price p and corresponding weekly supply q for a particular brand of diet soda in a city are shown in the figure. Use this graph to estimate the following supplies to the nearest 100 cases.

(A) What is the supply when the price is \$5.60 per case?

(B) Does the supply increase or decrease if the price is increased to \$5.80 per case? By how much?

(C) Does the supply increase or decrease if the price is decreased to \$5.40 per case? By how much?

(D) Write a brief description of the relationship between price and supply illustrated by this graph.

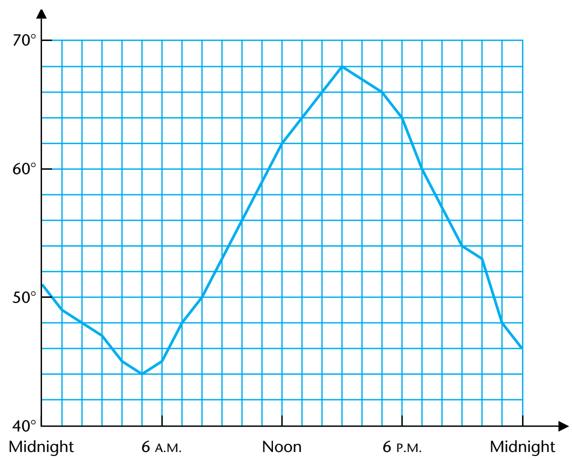


55. Temperature. The temperature (in degrees Fahrenheit) during a spring day in the Midwest is given in the figure. Use this graph to estimate the following temperatures to the nearest degree and times to the nearest hour.

(A) The temperature at 9:00 A.M.

(B) The highest temperature and the time when it occurs.

(C) The time(s) when the temperature is 49°.



56. Temperature. Use the figure for Problem 55 to estimate the following temperatures to the nearest degree and times to the nearest half hour.

(A) The temperature at 7:00 P.M.

(B) The lowest temperature and the time when it occurs.

(C) The time(s) when the temperature is 52°.

57. After extensive surveys, the marketing research department of a producer of popular cassette tapes developed the demand equation

$$n = 10 - p \qquad 5 \le p \le 10$$

where n is the number of units (in thousands) retailers are willing to buy per day at \$$p$ per tape. The company's daily revenue R (in thousands of dollars) is given by

$$R = np = (10 - p)p \qquad 5 \le p \le 10$$

Graph the revenue equation for the indicated values of p.

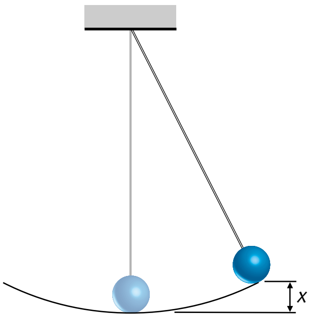
58. Business. Repeat Problem 57 for the demand equation

$$n = 8 - p \qquad 4 \le p \le 8$$

59. Physics. The speed (in meters per second) of a ball swinging at the end of a pendulum is given by

$$v = 0.5\sqrt{2 - x}$$

where x is the vertical displacement (in centimeters) of the ball from its position at rest (see the figure).



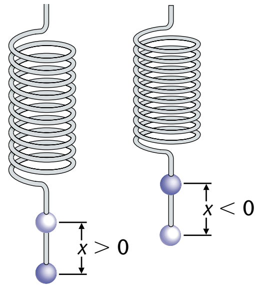
(A) Graph $v = 0.5\sqrt{2 - x}$ for $0 \le x \le 2$.

(B) Describe the relationship between this graph and the physical behavior of the ball as it swings back and forth.

60. Physics. The speed (in meters per second) of a ball oscillating at the end of a spring is given by

$$v = 4\sqrt{25 - x^2}$$

where x is the vertical displacement (in centimeters) of the ball from its position at rest (positive displacement measured downward—see the figure).



(A) Graph $v = 4\sqrt{25 - x^2}$ for $-5 \le x \le 5$.

(B) Describe the relationship between this graph and the physical behavior of the ball as it oscillates up and down.

Section 1-2 Using Graphing Utilities

- Graphing Utilities
- Screen Coordinates
- The Trace and Zoom Features

In the previous section, we sketched the graphs of equations by plotting points and then drawing by hand a smooth curve that passes through these points. Now we want to explore the use of electronic graphing devices to graph equations. The use of technology to aid in drawing and analyzing graphs is revolutionizing mathematics education and is the reason for this book. Your ability to interpret mathematical concepts and to discover patterns of behavior will be greatly increased as you become proficient with an electronic graphing device. If you have already used an electronic graphing device in a previous course, you can use this section to quickly review basic concepts.

Graphing Utilities

We now turn to the use of electronic graphing devices to graph equations. We will refer to any electronic device capable of displaying graphs as a **graphing utility.** The two most common graphing utilities are handheld graphing calculators and computers with appropriate software. You should have such a device as you proceed through this book.

We will discuss graphing utilities only in general terms. Refer to the manual or to the graphing utility supplement accompanying this text for specific details relative to your own graphing utility.